Write a **simplified** integral for the length of the curve $y = \int_1^x \sqrt{t^3+1}\,dt$ for $2 \le x \le 5$. **SCORE: _____ / 5 PTS**

Then use your calculator to find the length correct to 4 decimal places.

$\underbrace{\frac{dy}{dx} = \sqrt{x^3+1}}_{}$ ② OK IF INSIDE $\sqrt{}$ IN INTEGRAL

$$\int_2^5 \sqrt{1+\sqrt{x^3+1}^2}\,dx = \underbrace{\int_2^5}_{\frac{1}{2}} \underbrace{\sqrt{2+x^3}}_{1}\,\underbrace{dx}_{\frac{1}{2}} = \underbrace{20.6064}_{1}$$

An alarm clock has a randomized snooze feature. When it rings, if you hit the snooze button to silence it, the alarm waits a random number of minutes (X), then rings again. The probability density function for X is $f(x) = \begin{cases} k\sqrt{x}, & x \in [1, 9] \\ 0, & x \notin [1, 9] \end{cases}$. **SCORE: _____ / 9 PTS**

[a] Find the **exact** probability that the alarm rings again less than 4 minutes after you hit the snooze button. You will first need to find the value of k.

NOTE: You may check your final answer using your calculator, but you must show all work to get an exact (non-decimal) answer.

$$\underbrace{\int_1^9 k\sqrt{x}\,dx = 1}_{1} \longrightarrow \underbrace{\frac{2}{3}kx^{\frac{3}{2}}\Big|_1^9 = 1}_{1} \longrightarrow \frac{2}{3}k(27-1) = 1 \longrightarrow \underbrace{k = \frac{3}{52}}_{1}$$

$$\underbrace{\int_1^4}_{1} \frac{3}{52}\sqrt{x}\,dx = \underbrace{\frac{1}{26}x^{\frac{3}{2}}}_{1}\Big|_1^4 = \frac{1}{26}(8-1) = \underbrace{\frac{7}{26}}_{1}$$

[b] Find the **exact** median number of minutes before the alarm rings again.

NOTE: You may check your final answer using your calculator, but you must show all work to get an exact (non-decimal) answer.

$$\underbrace{\int_1^M \frac{3}{52}\sqrt{x}\,dx = \frac{1}{2}}_{1} \longrightarrow \frac{1}{26}x^{\frac{3}{2}}\Big|_1^M = \frac{1}{2} \longrightarrow ① \underbrace{\frac{1}{26}(M^{\frac{3}{2}}-1) = \frac{1}{2}}_{}$$

$M^{\frac{3}{2}} = 14$

$M = \underbrace{14^{\frac{2}{3}} \text{ OR } \sqrt[3]{196}}_{1}$

EITHER ONE OK

Find the **exact** length of the curve $y = \cos^{-1} x - \sqrt{1-x^2}$ from the point $(0, \pi - 1)$ to the point $(\frac{1}{2}, \frac{\pi}{3} - \frac{\sqrt{3}}{2})$. **SCORE: _____ / 8 PTS**

NOTE: You may check your final answer using your calculator, but you must show all work to get an exact (non-decimal) answer.

$$\frac{dy}{dx} = \boxed{-\frac{1}{\sqrt{1-x^2}}} - \boxed{\frac{1}{2}(1-x^2)^{-\frac{1}{2}}}\boxed{(-2x)} = \frac{-1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} = \boxed{\frac{x-1}{\sqrt{1-x^2}}}$$

1 1 $\frac{1}{2}$ $\frac{1}{2}$

$$\int_0^{\frac{1}{2}} \sqrt{1 + \left(\frac{x-1}{\sqrt{1-x^2}}\right)^2}\, dx = \int_0^{\frac{1}{2}} \sqrt{1 + \frac{x^2-2x+1}{1-x^2}}\, dx = \int_0^{\frac{1}{2}} \boxed{\sqrt{\frac{2-2x}{1-x^2}}}\, dx$$

2

$$= \int_0^{\frac{1}{2}} \sqrt{\frac{2(1-x)}{(1+x)(1-x)}}\, dx = \int_0^{\frac{1}{2}} \boxed{\sqrt{\frac{2}{1+x}}}\, dx = \sqrt{2}\int_0^{\frac{1}{2}} (1+x)^{-\frac{1}{2}}\, dx = \boxed{2\sqrt{2}\,(1+x)^{\frac{1}{2}}}\Big|_0^{\frac{1}{2}}$$

1 1

$$= 2\sqrt{2}\left(\sqrt{\tfrac{3}{2}} - 1\right) = \boxed{2\sqrt{3} - 2\sqrt{2}}$$

1

OK IF YOU CALCULATED $\frac{dy}{dx}$ AND SIMPLIFIED IT INSIDE THE $\sqrt{\ }$

Find the **exact** length of the parametric curve $\begin{matrix} x = 2 - 3t^4 \\ y = 5 + t^6 \end{matrix}$ for $t \in [1, 2]$. **SCORE: _____ / 8 PTS**

NOTE: You may check your final answer using your calculator, but you must show all work to get an exact (non-decimal) answer.

$$\frac{dx}{dt} = -12t^3, \quad \frac{dy}{dt} = 6t^5$$

$$\int_1^2 \boxed{\sqrt{(-12t^3)^2 + (6t^5)^2}}\, dt = \int_1^2 \boxed{6t^3\sqrt{4+t^4}}\, dt$$

$1\frac{1}{2}$ 1 1

$\boxed{u = 4 + t^4}$ 1, $t = 2 \rightarrow u = 20$, $t = 1 \rightarrow u = 5$

$\frac{du}{dt} = 4t^3 \rightarrow dt = \frac{1}{4t^3}\, du$

$6t^3\sqrt{4+t^4}\, dt = \frac{3}{2}u^{\frac{1}{2}}\, du$

$$= \boxed{\int_5^{20}}\, \boxed{\frac{3}{2}u^{\frac{1}{2}}\, du}$$ 1

$$= \boxed{u^{\frac{3}{2}}}\Big|_5^{20}$$ 1

$$= \boxed{20\sqrt{20} - 5\sqrt{5}}$$ 1

$$= 40\sqrt{5} - 5\sqrt{5}$$

$$= \boxed{35\sqrt{5}}$$ $\frac{1}{2}$